

General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

E85-10004

NASA Technical Memorandum 58260

(E85-10004 NASA-TM-58260-Pt-1) FUNDAMENTAL
REMOTE SENSING SCIENCE RESEARCH PROGRAM.
PART 1: STATUS REPORT OF THE MATHEMATICAL
PATTERN RECOGNITION AND IMAGE ANALYSIS
PROJECT (NASA) 44 p HC A03/MF A01 CSCI 05B G3/43 00004

N85-11417

Unclass

Fundamental Remote Sensing Science Research Program

Part II: Status Report of the Mathematical Pattern Recognition and Image Analysis Project

August 1984



National Aeronautics and
Space Administration



NASA Technical Memorandum 58260

Fundamental Remote Sensing Science Research Program

Part II: Status Report of the Mathematical Pattern
Recognition and Image Analysis Project

Richard P. Heydorn
Lyndon B. Johnson Space Center
Houston, Texas

August 1984

PREFACE

With the continuing technological evolution of remote sensing from space, it is evident that we benefit from the technology only to the degree to which we understand the information captured in the remotely sensed image itself. Images of the earth acquired from space vary according to the scene properties they portray. Images are dependent on the natural variance in radiance from the earth's surface, the effect of the atmosphere on the transfer of radiation, and the measurement capability of the sensor. What we can learn from orbital images depends on our ability to understand the transfer of electromagnetic energy from the earth's crust through the atmosphere, and the absorption, emittance, and reflectance characteristics of both organic and inorganic materials of the earth's surface. We must also be able to accurately register an orbital image, and the information contained therein, to its true location on the earth's surface. Thus, with an understanding of energy transfer from the target to the sensor and accurate procedures for geographical registration, we have the spectral and spatial attributes of an image that will allow us to infer the maximum amount of information from a scene. Some techniques that generate information from an image may be fundamental and generic in their application to the characterization of scene properties in all images. The development of generic techniques to advance our understanding of remotely sensed images represents an emerging, highly sophisticated science. The National Aeronautics and Space Administration, as an established sponsor of remote sensing technology research, has embarked on a specialized and continuing research program in fundamental remote sensing science. After an evaluation of major research needs, the agency has defined two significant projects:

1. Scene Radiation and Atmospheric Effects Characterization (SRAEC), and
2. Mathematical Pattern Recognition and Image Analysis (MPRIA).

In 1981, NASA solicited research proposals related to the two projects from both the NASA and external science community. After a competitive evaluation of submitted proposals, NASA selected approximately 35 investigations and awarded funding in 1982. The investigations of both research projects strive to improve our understanding of scene properties. The two projects can be differentiated by the basic approach underlying each. The SRAEC Project seeks to understand the fundamental relationship of energy interactions between the sensor and the surface target, including the effect of the atmosphere, to construct theoretical models predicting the radiance of the earth's surface. Model inversions can then be applied to interpret the information contained in a space-acquired image of measured radiance. Conversely, the MPRIA Project seeks to develop analytical techniques that group the radiance values contained in an image on a statistical basis to infer the properties of the scene, ultimately to understand the condition of the earth's surface. An important component of MPRIA lies in the development of technique for image georegistration and recognition of texture. The information associated with spatial patterns, or texture, of radiance in an image may contribute substantially to the inference of scene properties.

The Fundamental Remote Sensing Science Research Program supports the long-term goals of NASA in two significant ways. First, the techniques developed through the program enhance our ability to learn more about the physical and biological processes of our planet from space acquired data. Second, the results of the investigations contribute to a base of scientific understanding needed to support the planning of new and effective sensors and flight

programs. This report is submitted to describe the Fundamental Remote Sensing Science Research Program and the progresses made since its initiation approximately two years ago. The report is represented in two parts. Part I provides the status of the Scene Radiation and Atmospheric Effects Characterization Project, primarily reflecting research results presented at the Second Annual Workshop for investigators held at Colorado State University in Fort Collins, January 9-11, 1984. Part II provides the status of the Mathematical Pattern Recognition and Image Analysis Project, which consists of current results and information summarized from the proceedings of the NASA Symposium on Mathematical Pattern Recognition and Image Analysis held June 1-3, 1983. (See Note 1)

By the end of 1984, the Land Processes Branch of the Earth Science and Applications Division, Office of Space Science and Applications, will announce a new opportunity for research in this continuing program. Topics for the solicitation of research will be defined in the months ahead and will be based on the outgrowth of results of present investigations and the fundamental research needs of other NASA Programs that incorporate remote sensing for earth observations.

M. Kristine Butera
Program Manager
Fundamental Remote Sensing
Science Research
NASA Headquarters
Washington, DC

(Note 1) Guseman, Jr., L. F. 1983. Proceedings of the NASA Symposium on Mathematical Pattern Recognition and Image Analysis, June 1-3, Johnson Space Center, Houston, Texas. Contract NAS 9-16664. Texas A&M University, Department of Mathematics, College Station, Texas.

TABLE OF CONTENTS

	Page
Executive Summary	vii
1.0 The Role of Mathematical Pattern Recognition and Image Analysis Within the Fundamental Remote Sensing Science Research Program	1
2.0 Summary of MPRIA Investigations	6
2.1 Preprocessing	8
2.2 Digital Image Representation	11
2.2.1 Spectral Representations	13
2.2.2 Spectral-Spatial Representations	14
2.3 Object Scene Inference	18
3.0 Summary of Accomplishments	20
3.1 Preprocessing	20
3.2 Digital Image Representation	21
3.3 Object Scene Inference	24
4.0 Schedule of Events	26
5.0 Publications	26

PRECEDING PAGE BLANK NOT FILMED

EXECUTIVE SUMMARY

The Mathematical Pattern Recognition and Image Analysis (MPRIA) Project is concerned with basic research problems related to the study of the earth from remotely sensed measurements of its surface characteristics. The program goal is to better understand how to analyze the digital image that represents the spatial, spectral, and temporal arrangement of these measurements for purposing of making selected inferences about the earth.

Initiated in July of 1982, the MPRIA project contains investigations from twelve universities and research organizations and three NASA Centers. These investigations are grouped in research categories called Preprocessing, Digital Image Representation, and Object Scene Inference.

Preprocessing research is concentrating on methods for registration and rectification of digital images. By registration we mean the process by which two or more images are aligned so that the same point on the ground is represented by the same pixel in each image. Rectification is the process by which an image is brought into alignment with a map.

Given a digital image of some scene, a digital image representation is a mathematical transformation (a model) of the image to a form that is useful for making an inference about the scene. Often these representations are mathematical descriptions of some characteristics of the earth's surface such as "texture" or "shape". Other times these representations are more abstract. For example, they may represent the image pixel values from a class of materials in the scene by a class conditional probability density.

Methods for developing a specific conclusion about the area being imaged (the scene) are studied in the object scene inference investigations. Generally, these conclusions can be related to mapping, inventory, or possibly condition assessment questions.

The MPRIA project along with the Scene Radiation and Atmospheric Effects Characterization (SRAEC) Project make up the Fundamental Remote Sensing Science Research Program. Investigations in SRAEC are aimed at understanding the physical scattering and absorption of electromagnetic radiation from the earth that give rise to the digital image that is studied in MPRIA.

1.0 The Role of Mathematical Pattern Recognition and Image Analysis Within the Fundamental Remote Sensing Science Research Program

From the remote sensing point of view, the measurement of electromagnetic energy reflected from the Earth's surface within various wavelengths of the spectrum provides us with a way to map a natural or cultural portion of the Earth's surface, called the scene, to an electronic digital image. Learning how to analyze the scene from the information that is preserved by this mapping is the theme of the Fundamental Remote Sensing Science Research (FRSSR) program. The FRSSR program has chosen to begin this learning process by dividing the studies between two program projects called Scene Radiation and Atmospheric Effects Characterization (SRAEC) and Mathematical Pattern Recognition and Image Analysis (MPRIA).

Since both SRAEC and MPRIA are concerned with inferences about the scene, how then do they differ? Before we attempt to give a general answer to this question, we will illustrate one possible difference with the following example.

Let us suppose that we wish to determine the average leaf area, which we denote by X , of a given species in some selected region. The average leaf area is the one sided (or projected) area of all the leaves in a plant canopy (for a given species) divided by the number of pixels in the area covered by the canopy. This, of course, would be the average leaf area at the time the measurements are taken. Furthermore, let us suppose that from our SRAEC studies we have determined that for the j th species $j = 1, 2, \dots, M$, the

model, represented by the function G_j , that relates leaf area to the sensor radiance values, denoted by R , is

$$X = G_j(R)$$

Presumably this model would be derived from the scattering and absorption properties of both the plant canopy and the atmosphere. In effect we have inverted a mapping from the scene to the image to give a determination of the scene property of interest to us, viz. average leaf area. However, to apply this model it would seem that we must know in advance that we are observing a given species. We therefore need yet another model, also based on our observed radiance values, to tell us which species is being observed. This second model is the kind of model that is studied in MPRIA.

If the species can be well separated from the radiance values then a classification model may be a good choice. The classification model is a partition of the space of all radiance values into the sets A_1, A_2, \dots, A_M . If $R \in A_j$ we decide that the image pixel having this radiance is associated with species j . Having classified each pixel, we simply add the leaf area of all pixels from species j (this, of course, would be the total leaf area for species j) and divide by the total number of pixels from that species to calculate the desired average.

If the species can not be well separated, and this can be determined by comparing the classifications with ground features, then the classification model may be inappropriate since the mistakes made in attempting to classify a pixel could introduce a substantial error in the final leaf area determination. For this case a mixture model is an alternative choice. In this approach the

unconditional probability density of the radiance values, f , is used to determine the species conditional densities, f_j , using the model

$$f(R) = \sum_{j=1}^M \text{Pr (Species } j) f_j(R)$$

(If the densities f_j are members of a known identifiable family of densities then it is possible to uniquely find each f_j from knowing f .) The average leaf area for species j is then gotten from the SRAEC leaf area model and the MPRIA mixture model as follows:

$$E(X|\text{Species } j) = \int G_j(R) f_j(R) dR$$

Often more than one species is represented by one pixel due to the coarse resolution of the sensing instrument. When this is true neither of the above models (classification or mixture model) holds. If the radiance values from a given species can be represented by a weakly stationary random process along any transect in the image, then a texture model may apply. The idea is to model the covariance function of the radiance values along a transect as a mixture of species conditional covariance functions. The covariance function for a weakly stationary process is a measure of the relatedness of two pixel radiance values taken Z units apart and it does not depend upon where the pixels are selected along the transect. A possible form for the model is

$$h(Z) = \sum_{j=1}^M \left((u_j - u)^2 + h_{jj}(Z) \Pi_{jj}(Z) \right) \text{Pr (Species } j) \\ + \sum_{\substack{i=1 \\ i \neq j}}^M \sum_{j=1}^M h_{ij}(Z) \Pi_{ij}(Z) \text{Pr (Species } j)$$

Here h is the above mentioned covariance function (the covariance of the mixture) and h_{ij} is the species conditional covariance between species i and

species j . The function Π_{1j} is the probability of observing a radiance value from species j that is Z pixels away from a pixel of species i . The constant u_j is the mean for the j^{th} species and u is the mean of the mixture. While not discussed, the conditional covariances and conditional probabilities have been "smoothed" by the kernel that defines the sensor and atmosphere. When this model applies we have, in a statistical sense, the ability to represent the total radiance from a pixel in terms of its species component radiances. The average leaf area for species j (assuming the radiance values are normally distributed) is

$$E(x|\text{Species } j) = \int G_j(R) \frac{1}{\sqrt{2\pi h_{jj}(o)}} \exp\left(-1/2 \frac{(R-u_j)^2}{h_{jj}(o)}\right) dR$$

This example is illustrative of the distinction between SRAEC and MPRIA. In SRAEC we are concerned with developing a theory to explain the way the properties of the earth's surface reflect and emit electromagnetic radiation through an atmosphere column to a remote sensing device. Presumably such studies would lead to models which can be "inverted" to estimate values of scene properties. In MPRIA we are concerned with developing yet other models to understand scene properties that combine various representations or models of the given data to develop an inference about the scene. The leaf area example was a case where radiance values from the scene were transformed to leaf area values per species by one (SRAEC) model and also transformed to conditional density functions or a partition of the scene measurements using yet another (MPRIA) model. Both models were then used to estimate (infer) the average leaf area. We refer to these models, in MPRIA, as Digital Image Representations.

The idea that an image is, in some sense, an organization of several representations is imbedded in the modern approach to image understanding. A multiple level of organization is generally assumed. At the lowest level the image is viewed in terms of representations that describe primitive structural elements such as lines, corners, edges, or even texture. These representations are an attempt to segment the image into elements which can be combined at another level of organization to develop representations of scene structure or shapes. By knowing scene structure we may be able to infer which parts of the image are, for example, the roads, the rivers, the cities, the forests, etc.

The point is that these digital image representations along with other inductive spectral models of the physical properties of the earth are, in a sense, the building blocks of inference and both SRAEC and MPRIA are contributing to the theory of object scene inference. In MPRIA we have concentrated on extending the theories that have been proposed in computer science, photogrammetry, statistics, mathematics and geography to develop representations which we feel have a bearing on our problems. In SRAEC we have used ideas from physics, meteorology, plant physiology, and other disciplines to develop our representation theories founded on physical processes.

2.0 Summary of MPRIA Investigations

Above we pointed out that study areas related to digital image representations, and, object scene inference are addressed within the MPRIA program. There is yet another area of study which we have called Preprocessing. By preprocessing we mean all preparatory manipulations of the image data that precede the representation phase. Presently all our preprocessing studies are related to the registration or the rectification of images.

We will now summarize some of the accomplishments within these three research areas. For a detailed technical discussion of the accomplishments up to June 1983, the reader should refer to "Proceedings of the NASA Symposium on Mathematical Pattern Recognition and Image Analysis," June 1-3, 1983 (c.f. footnote in the Preface). There will be another proceedings published after the 1984 June symposium, which will present a detailed documentation of the second year's effort.

Our summary will point out the primary individual who is responsible for the investigation by underlining the last name in the following text. The names of the researchers along with their investigation titles are given in Table 1.

TABLE 1.

MPRIA INVESTIGATIONS

<u>INSTITUTION/INVESTIGATOR</u>	<u>INVESTIGATION</u>
<u>TEXAS A&M</u>	
L. F. Guseman & L. L. Schumaker	Spline Classification Methods
E. Parzen	Quantile Data Analysis of Image Data
W. B. Smith	Discrimination Relative to Measures of Non-normality

MPRIA INVESTIGATIONS (Continued)

<u>INSTITUTION/INVESTIGATOR</u>	<u>INVESTIGATION</u>
H. J. Newton	Repeated-Measures Analysis of Image Data
<u>U OF TEXAS</u>	
C. N. Morris	An Empirical Bayes Approach to Spatial Analysis
<u>PURDUE</u>	
E. M. Mikhail	Simulation Aspects in the Study of Rectification of Satellite Scanner Data
<u>LNK</u>	
L.N. Kanal	Analysis of Subpixel Registration Accuracy
<u>U OF MARYLAND</u>	
L. S. Davis	Image Matching Using Generalized Hough Transforms
<u>U OF HOUSTON</u>	
C. Peters	Mixture models for Dependent Observations
<u>HUNTER COLLEGE</u>	
A. H. Strahler	Relating Spatial Patterns In Image Data to Scene Characteristics
<u>SRI</u>	
G. B. Smith	Shape from Shading: An Assessment
<u>U OF KANSAS</u>	
K. S. Shanmugan	The Influence of Sensor and Flight Paramter on Texture in Radar Images

MPRIA INVESTIGATIONS (Continued)

<u>INSTITUTION/INVESTIGATOR</u>	<u>INVESTIGATION</u>
<u>U OF CALIF. AT SANTA BARBARA</u>	
W. R. Tobler	Fractal Models of Texture
<u>JSC</u>	
R. P. Heydorn	Estimating Location Parameters in a Mixture Model
D. W. Scott/Rice University	Multivariate Density Estimation and Remote Sensing
A. G. Houston	Estimation with Classifier as Auxiliary Variable
R. F. Gunst/Southern Methodist University	Crop Area Estimation Based on Remotely Sensed Data with an Accurate but Costly Subsample
<u>JPL</u>	
J. P. Held	SAR Speckle Noise Reduction Using Wiener Filter
M. Naraghi	Autoregressive Models for Use in Scene Segmentation
<u>NSTL</u>	
D. D. Dow	Progress in the Scene-to-Map Registration

2.1 Preprocessing

Recognizing that the sensor pointing errors are a major source of error when attempting to register or rectify images Mikhail has developed a general sensor/platform error model. From the model we can compute the ground position of the image pixels given the values of the parameters in the model. Registration or rectification accuracy can therefore be

studied by introducing errors into these parameters and thereby obtain an understanding of how accurately these parameters need to be estimated. Presumably, these parameters can be estimated from ground control points, and so, e.g., one can study rectification/registration errors as a function of the number, placement, and location accuracy of ground control points. The model is now being used to rectify two frames of MSS Landsat data from scenes over Kansas and Louisiana and to study the effect of varying the number of control points on rectification accuracy. Some early results suggest that for these scenes using fifteen (15) control points RMS errors are about 70 meters. With an additional fifty (50) control points only a minor improvement (about 3-4 meters) was noted in the RMS error. Dow has attempted to rectify this same data using a mapping developed from the ground control points. This mapping was not based on the use of a sensor/platform error model.

Kana is considering the problem of registering two images to subpixel accuracy. This effort is concerned with methods for estimating registration accuracy at the subpixel level. Most recently, the work has followed two main approaches. Both of these approaches are aimed at accurate estimation of edge positions. These estimates will then be used to match lines between a reference image and a sensed image to provide a registration. First, the work on determining the subpixel location of an edge given the set of observed edge pixels has been extended. In the first year of the study, procedures were developed for estimating the error in subpixel edge position given the correct digitization of an edge. Parts of that work have been extended to allow for errors in the detection of edge pixels. For small numbers of incorrectly detected edge pixels, a description of the possible erroneous digital lines has been

developed. Procedures for computing various types of error estimates using this set have been developed and are currently being tested. The geometric structure, in polar coordinate space, of the regions corresponding to the various digital lines has been examined. Various results relevant to the analysis of line position errors for larger numbers of incorrectly detected edge pixels have been developed using this geometric information.

Procedures for directly estimating straight edge subpixel positions given the approximate location of the edge have been developed. This work uses polynomial fitting explicitly incorporating the constraints of straightness, known approximate position of edges, and known orientation of edges. The methods are currently being tested on the LANDSAT data. The subpixel estimates will be used directly for estimating subpixel edge positions as well as estimating edge pixels to use as input to an algorithm for computing subpixel edge location from the edge pixels. These results will be compared to determine the most suitable algorithm.

When two images have been acquired over the same area on the ground but spaced over a large time span or have been acquired by two different sensors, the images often cannot be reliably matched, or registered, using traditional registration algorithms based on either intensity or edge correlation. For such cases Davis has proposed the following approach to registration:

- (1) The images are first segmented into regions that have distinct structural or textural properties.

- (2) The segments are analyzed by an expert vision system that has available to it detailed, but highly specific, models for the entities that will appear in the scene as viewed through a particular sensor. The result of this analysis is that the regions produced by the segmentation are labeled (i.e., classified); furthermore, the initial segmentation is re-analyzed and refined.
- (3) The final interpreted regions are used by a matching algorithm to register the images.

In the area of segmentation, algorithms for extracting both compact and elongated regions from images have been developed. Also, robust representations of image texture based on correlation of ranked data have been developed.

Finally, this registration research has developed matching algorithms for image registration that are capable of utilizing both the structural information concerning the appearance of image regions and the semantic information computed by the expert vision system to register images. These algorithms involve Hough transform techniques for estimating the parameters of the registration. (Hough transforms are discussed further on page 18.)

2.2 Digital Image Representation

A digital image that is produced in remote sensing applications is a two dimensional array of, generally, vector valued measurements (radiance values). If the array is associated with "one look" of the sensor, then an

element (one entry) of the array is a vector of as many measurements as there are measurement channels on the sensor. For example on the TM sensor of Landsat 4 and Landsat 5 there are seven (7) channels. The array in this case would have elements which are 7 dimensional vectors. Often false color images are made from the digital image. In this case one might pick three channels from the seven and assign a primary color intensity to a channel value to produce the color image.

If, however, the array is obtained by registering a time sequence of looks then an element of the array is a vector which is a sequence of measurements. Returning to our TM sensor example, imagine that the sensor looks at the same area on the ground at time, t_1 , time t_2 , etc. The first seven entries in the vector are the measurements at time t_1 the next seven are the measurements at time t_2 , etc.

The fact that in remote sensing we are often dealing with arrays that are vector valued, as apposed to a panchromatic image, means that even when we ignore the spatial arrangement of the elements - i.e., treat the elements independently - we can obtain a considerable amount of information about the scene.

In discussing the representation studies we will single out those which are spectral and those which are spectral-spatial in nature. (The word spectral is derived from the fact that each vector element is a measurement from a wavelength interval in the electromagnetic spectrum.) For the spectral case we can take a random sample of array elements to develop the representation. For the spectral-spatial case we must keep track of the location of each element in the array or at least keep track of locations in a local neighborhood of a given element.

2.2.1 Spectral Representations

These studies have concentrated on probability density function representations for the variety of classes that are represented in the data.

As an exploratory tool to understand the structure of the density functions, Scott has developed a four dimensional color computer graphic program in which three dimensions in the data are each represented by a color and the fourth dimension is represented by time, i.e., a sequence of images is produced. He has also developed a new density estimator called the average shifted histogram that can be computed many times faster than the well known kernel estimator.

Guseman and Schumaker are using B-spline theory to estimate density functions with the aim of computing a Bayes classification boundary. Since the Bayes boundary turns out to be the set of zeros of a linear combination of splines, this approach leads to a computationally efficient method, at least in the case of univariate density functions. The technique is being extended to the multivariate case where the Bayes boundary is a "zero contour or zero surface." Given the Bayes boundary, the method also leads to a computationally efficient estimate of the probability of misclassification. A B-spline is a piecewise continuous polynomial that satisfies certain regularity conditions at the break points (knots). The "B" refers to the fact that these splines are used as basis functions in the approximation process.

The unconditional probability density function of the measurements, which will be called the mixture density, in an image can often be described as a linear (convex) combination of conditional densities. A

given conditional density describes the measurements from one and only one class of materials on the ground. When these conditional densities are members of a known identifiable family (and real data suggests that this is often the case) then the conditional densities can be uniquely derived from the unconditional density. The representation of the mixture density as this linear combination of conditional densities is called a mixture model. Heydorn is taking this mixture model approach for determining representations for the image data. The current studies are concentrating on families whose members are translates of functions whose Fourier or Laplace transform is a rational function. Exponential, gamma, and beta families are examples that have these characteristics. A method has been developed based on a Caratheodory representation theorem for determining the number of conditional densities in the mixture and the individual translation values. More recently studies have concentrated on constrained B-spline estimation methods to derive estimators of these quantities.

2.2.2 Spectral-Spatial Representations

One of the properties of a scene that is presumably used by human image interpreters is texture. One possible representation for texture is to model it using a spatial statistics model also called the random field model. Naraghi has proposed the random field model for texture with the aim of segmenting a scene into texture types. In this model an array element is modeled as a linear combination of surrounding array elements (i.e., an autoregressive formulation). Having modeled each texture class in this way, a Bayesian classification scheme is then used to segment the scene into texture class groups.

Shanmugan has also proposed an autoregressive texture model. In his model, however, an array element is modeled as a linear combination of piecewise continuous functions centered on surrounding array locations. The philosophy behind this approach says that texture is made up of a random arrangement of primitive elements. The primitive elements are the above piecewise linear functions. Texture models of this kind are then used to define a texture edge for segmenting the scene.

Another approach for constructing a texture model based on "randomized" primitive elements has been taken by Tobler. Tobler considers a fractional Brownian motion model, which is a form of a fractal model. This model can be derived by operating on a brownian motive process using a certain smoothing kernel.

Whenever a spatial statistics model for texture is proposed for a vector valued array, massive data management and computational problems can occur. Newton is studying models based on parallel transects with the idea of only using the correlation along the transect.

Texture appears to be a phenomenon that depends upon the spatial resolution of the instrument that is viewing the scene. At fine resolution often one type of texture is visually apparent; but, at another resolution another texture type is apparent. The effect of resolution in various scene models is being studied by Strahler. He has begun by introducing a concept of local variance which can be estimated from image values by starting with a fine resolution image, and while successively degrading the resolution, computing the average variance from radiance values in a moving window. Over forested areas these

studies have shown that the local variance peaks when the resolution of the instrument approximates the average size of tree crown. Over agricultural areas the peaking tends to occur around resolutions related to average field sizes. For residential areas where there is a variety of object sizes in the scene, the curves of local variance versus resolution, appear to broaden out; but, each curve still has a single peak.

Another application of a spectral-spatial representation is considered by Peters. In this study a mixture model that uses the local spatial properties is proposed. The array values along a given row are placed in groups of varying sizes. The idea here is that one would construct these groups from elements belonging to a single agricultural field or belonging to some homogeneous region in the scene. And, since fields or homogeneous areas can vary in size, the sizes of the groups are treated as random observations. This model has been derived under the so-called exchangeability hypothesis which implies that the probability density functions of the array elements in the group is independent of the location of the elements. Approximations to groups that have a Markovian structure are considered using the exchangeability hypothesis.

Parzen is exploring the use of statistics based on quantiles as a possible method for representing the image data. In this approach a square neighborhood of an array element is first defined and then a sample quantile function is computed using the values in the neighborhood. From this quantile function a number of statistics are derived. Some of these statistics are the median, the interquantile range, the information quantile function, and ratios and differences of these quantities. The idea behind this approach is that one can derive good discriminating

features for a ground class from the quantile function. Empirical studies with agricultural data suggest that features derived in this way can separate many crop types and natural vegetation. Besides being useful for developing feature representations for the data, these quantile methods can also be used to test certain hypotheses about the form of the class conditional density functions.

Texture is one property that can be used to describe a scene, and shape is another (c.f. page 18). Smith (SRI) is considering the problem of reconstructing 3D surface shape from 2D imagery. He showed what 3D shape information is available in an image although his information is insufficient to allow direct reconstruction of the 3D surface shape. He has therefore approached the problem using the predict and verify paradigm. Using DTM (Digital Terrain Model) data (obtained from say, matching a stereo pair) he constructed a representation of the surface. This representation is used to predict the shape information that can be obtained from the image directly. Verification allows him to assess where the surface model is correct and where it needs to be adjusted. (It also allows assessment of the quality of the DTM data.) Large surface models are computationally expensive, while multipatch models have joining problems. He has found computationally effective means for creating large surface models of natural terrain. At present he is using these surface models for image prediction and is investigating methods for adjusting the model at those places where it incorrectly predicts the image information.

Held is studying the problem of removing speckle in synthetic aperture radar (SAR) images using Weiner filter theory. This filter is

determined from the power spectral density functions (psd) of the scene and the noise. (The psd is the Fourier Transform of the autocorrelation function.) The noise psd in this case is modeled by a delta function plus a band limited white noise term.

2.3 Object Scene Inference

The classification of objects in the scene based on the radiance values is often done to produce a labeled map of the scene or to obtain an inventory of a class of objects in the scene. Classifications based on linear discriminant functions are appealing since the number of parameters that needs to be determined is small in comparison to other discriminant functions. If, however, a linear function is assumed when in fact the correct discriminant is nonlinear, then less than optimal performance can be expected. Smith (Texas A&M) is studying the robustness of linear discriminants when the linearity assumption is violated. To study robustness he has chosen a model for the class conditional densities that is a mixture of normals. With this model he computes the Bayes risk and compares it with the risk when a linear discriminant is used.

The Bayes classification rule is based on the posterior probability for each class in the scene. It is sometimes possible to determine these posterior probabilities from the spectral values alone without knowing the class prior probabilities. The general class of methods that estimate the posteriors in this way are often called empirical Bayes methods. Morris is considering empirical Bayes methods to derive the label for a given pixel by considering the labels of surrounding pixels. The idea is based on the notion of an affinity matrix and the Stein

shrinkage estimator. The affinity matrix is used to smooth posterior probability estimates based on neighboring pixels. These smoothed estimates along with the original estimates are linearly combined to form the Stein shrinkage estimate. Depending on the variances of the smoothed and unsmoothed estimates, the Stein shrinkage estimator will tend to "shrink" toward one estimate or other.

One way to inventory a scene is to first classify the objects in the scene and then simply count the objects which have been classified to a given class. The method, however, can give a biased estimate when classification errors are committed. Given a small sample of ground truth observations, one can regress an estimate based on this sample against the classifier-derived estimate to obtain an unbiased estimate. This kind of a regression estimator can have a better sampling efficiency than does the estimator just based on the ground truth random sample. When the regression is linear the sampling efficiency can be measured by the correlation between the two estimates. Houston is studying regression estimators of the general kind. He is also comparing this regression estimator with the so-called calibration estimator, where the order of the regression is reversed, the ratio estimator, and the stratified estimator. This latter estimator is not a regression estimator. Comparisons are being made among these estimators to understand the situations where a given estimator is best. The ratio estimator projects the classifier-derived value to a ground truth value based on ratio of estimates of these quantities. The stratified estimator uses the classifier to create two strata. A weighted sum of stratum ground truth derived estimates is then used as the final estimate.

Along these lines, Gunst is investigating the errors-in-variables model for relating classifier-derived estimates and ground observations. This approach further assumes errors in the ground observations and requires an additional parameter. This parameter takes the form of the ratio of the variance of errors in the ground observations to the variance of the classifier-derived estimates. The latter variance is due to training sample variation.

3.0 Summary of Accomplishments

The MPRIA program project has been continuing for approximately 1-1/2 years. During that time each investigator has made substantial progress in establishing a theoretical foundation for his work. Many of the investigators have been or are now beginning to test their theories on remotely sensed data. In a few cases computer programs have been developed which could be transported to other researchers in the country interested in applying these ideas in their studies.

We now summarize some of the specific accomplishments that have occurred in the program.

3.1 Preprocessing

By using a set of control points which can be located in both the image and on a map of the ground being imaged, it is possible by methods of interpolation, to approximately rectify the image to the ground. The control points, e.g., are used to determine the coefficients in the polynomials, or splines, or whatever interpolation function is used. This approach is an attempt to approximate the function that created the mapping between ground and the image without explicitly modeling the

imaging device (the sensor and the sensor platform). It is therefore worthwhile to ask if significant improvement in rectification can be achieved by an explicit model of this kind. This model would be a parametric model that models as closely as possible the geometric and physical processes that produced the imagery. Such a model is being developed for the MSS and TM sensors used in the Landsat series by Mikhail. This model is now to the point where it can be used to rectify imagery using control points to estimate the parameters in the model. In a preliminary study where two MSS images were rectified (one scene being in Kansas and the other in Louisiana) the RMS error was about 68 meters for the Kansas site and 72 meters for the Louisiana site, using 15 control points to estimate model parameters. The errors dropped slightly (3 to 4 meters) when the number of control points was increased to 81 for the Kansas site and to 70 for the Louisiana site. With 15 control points, it was therefore possible to rectify the images to less than 1 pixel RMS error.

3.2 Digital Image Representation

Many natural and cultural scenes contain a large number of objects which when taken individually may be hard to recognize, but when considered in groups they seem to have characteristic properties. Texture appears to be this kind of a phenomenon. If one could segment a scene by forming texture groupings, then it may be possible to focus on just the parts of the scene that are of interest.

While the definition of texture is still controversial, it would seem that it is, roughly, some kind of repetitive arrangement of elemental shapes (primitives) over a given area. Since the arrangement seems to

have a "random quality", it is perhaps natural to try to represent texture by some kind of random process. This random process need not reproduce each radiance value from the scene but rather lead to a model which maps all the radiance values in some region to a few numbers which can be used to discriminate texture types.

In MPRIA we are in the early formative stages of understanding texture in remotely sensed images. Our first attempts have adapted some concepts of a random process. Shanmugan, for example, tries to model texture using a few primitive functions that occur randomly and are weighted and added to form the random process. It is, however, not clear from his model how one might select the set of primitive functions or if the selection should depend upon the texture type being modeled. Strahler's work may shed some light on the question. He has developed an approach for estimating an optimal size for a resolution cell in a texture which has a bearing on the selection of suitable primitive functions. By knowing how this size varies in an image one could adjust the size to optimize texture identification. Tobler's work with fractals also suggests that a random process can lead to a model for texture, as evidenced by the fact that such models produce very realistic simulations of natural terrain.

Whereas texture is a property that relates to a collection of objects, where the arrangement of the objects is the important characteristic, shape is a property that relates to individual objects. In MPRIA we have looked at certain fundamental questions related to shape. First of all, Smith (SRI) has considered the problem of being able to recover a three dimensional object from only shadow information in a two dimensional

image. While a lot can be learned just from observing the shadows, the object cannot be totally reconstructed from just that data. Smith is therefore considering using a model to predict the shape of the object in hopes of being able to add "a priori" information in this reconstruction process. Davis is also using models to determine certain shapes in an image. In this approach, a contour (a two dimensional outline of an object) is taken from one image and used as a model to find a similar contour in another image. This is done by translating, rotating and perhaps topologically distorting the contour until it matches with a contour in the second image. The actual approach is called the generalized Hough transform.

Many of the inferences one would like to make about the scene are based on a statistical argument. The example in Section 1.0 where we estimated average leaf area per species is one illustration. These kinds of inferences often depend upon the probability densities of the classes of interest in the scene. Representations of the scene in terms of these class conditional densities are therefore useful. One way to derive this kind of a representation is to obtain a sample of ground truth observations from each class in the scene and derive the class conditional densities using these samples. The problem with this approach is that the ground truth sample is often not available. There are, fortunately, some interesting theorems in statistics which suggest that one might be able to derive the class conditional distributions without having the ground sample. These theorems are known as the identifiability theorems for mixture models. The applicability of these theorems has been tested using remotely sensed data from agricultural sites and have been found in many situations to be good approximations to reality.

To apply a mixture model, one must know in advance the family of densities that describes each of the class conditional densities. One must also be able to estimate the number of component densities in the mixture. With regard to the first problem, Scott has developed a computer graphic approach that allows one to display density contours of up to four variables. By looking at sample data it is often possible to spot certain characteristics of the multivariate nature of these densities. Often dominant directions in the variable space suggest that certain projections or other transformations are warranted. This programming has reached the stage where the computer software could be available to other researchers interested in multivariate density representations. Heydorn is addressing the problem of estimating the number of components in the mixture for families of densities that appear to typically represent some of the skewed nature of real class conditional densities. He has been able to show that for several families of univariate densities the number of components can be computed. The extension to multivariate densities has as yet not been attempted.

3.3 Object Scene Inference

One of the more basic applications of remotely sensed data is to use it to increase the precision of a given sample survey. An example is as follows:

The objective is to estimate the number of aspen trees in a given forest. From a random sample in which a survey team counts the number of aspen trees growing in randomly selected tracts, one can obtain an estimate within a certain precision. By using in addition classifications of tree types over many more tracts, it is possible

to derive an estimate of the same quantity using just the remotely sensed data. The first estimator can be considered as unbiased, but with large variance (low precision) since only a few tracts are used. Of course by increasing the number of tracts used the precision would increase, but this is generally an expensive option. The second estimator has a small variance, since the number of tracts used is large, but since the classifications are seldom error free, the estimator is generally biased. By developing a regression using these two estimators, it is possible to derive another estimator which captures the unbiasedness of the first estimator and some of the low variance properties of the second estimator.

While general problems of this type have been studied in statistics for some time, there are still a number of unresolved theoretical questions related to this remote sensing application. Houston is considering this type of a regression problem along with other estimators; specifically, the poststratified and the ratio estimators, in an attempt to understand their behavior in terms of the classifier design and the probability distribution structure of the remotely sensed data. By studying several simulated cases he has found, for example, that when one compares the regression estimator with the stratified estimator the former is generally better (has lower variance) unless the underlying class conditional probability distributions are highly overlapped (the classes are difficult to discriminate from the remotely sensed data). One of the goals of these studies is to develop a theory of classifier design which will lead to highly efficient (low variance) area estimation models and to develop estimation methods for the parameters in these models.

4.0 Schedule of Events

The major technical meetings in the MPRIA program are the symposiums which are held each June and the workshops. The first symposium was held June 1-3, 1983, at the NASA Johnson Space Center, Houston, Texas. The proceedings from that symposium have been published. In 1983, there were also two workshops for the purpose of reviewing the program. One was held January 27-28, and was devoted to the more mathematical and statistical investigations in the program. The other was held February 3-4, and covered the pattern recognition studies. This year, in preparation for the program renewal, the workshops will be devoted to special topics which are intended to explore new possible study areas that should be included in future work. One workshop on multivariate spline models was held February 15-16, 1984. Another on computer graphics is being planned for this spring or early summer. Other workshops are also being planned.

5.0 Publications

The following is a list of publications resulting from MPRIA studies.

Journal Articles

R. F. Gunst, Toward a Balanced Assessment of Collinearity Diagnostics, American Statistician, 38, (to appear, May 1984).

R. F. Gunst and R. L. Eubank, Regression Diagnostics and Approximate Inference Procedures for Penalized Least Squares Estimators, submitted to JASA.

M. Y. Lakshminarayan and R. F. Gunst, Estimation of Parameters in Linear Structural Relationships: Sensitivity to the Choice of the Ratio of Error Variances, Biometrika (to appear in 1984).

M. Naraghi, Scene Segmentation Using Autoregressive Models, submitted to IEEE Trans., Pattern Analysis and Machine Intelligence.

L. L. Schumaker, C. K. Chui and R. H. Wang, On Spaces of Piecewise Polynomials with Boundary Conditions. II.Type-1 Triangulations, in Second Edmonton Conference on Approximation Theory, Ditzian et al., eds. CMS Vol. 3, AMS, Providence, 1984, 51-66.

L. L. Schumaker, C. K. Chui and R. H. Wang, On Spaces of Piecewise Polynomials with Boundary Conditions. III.Type-2 Triangulations, in Second Edmonton Conference on Approximation Theory, Ditzian et al., eds. CMS Vol. 3, AMS, Providence, 1984, 67-80.

D. W. Scott, Frequency Polygons: Theory and Application, submitted to JASA.

K. S. Shanmugan, Influence of Sensor and Flight Parameters on the Textural Properties of Radar Images, to be published in IEEE Transactions on Geosciences and Remote Sensing.

W. B. Smith and M. W. Riggs, Akaike Information and Missing Multinomial Data, submitted to Statistical and Probability Letters.

W. B. Smith and K. K. Moore, Distribution and Simulation of the Rank Transform Method, submitted to Comm. in Stat. B.

P. Spector and H. J. Newton, Box's Correction for Toeplitz Error Covariance Matrices, submitted for publication.

G. Terrell and D. W. Scott, Oversmooth Nonparametric Density Estimates, submitted to JASA.

Symposia and Conferences

M. A. Calabrese and R. E. Murphy, "Improving Our Understanding of the Remote Sensing Process," with Presentation at the 1983 IGARSS Symposium.

L. S. Davis and A. Rosenfeld, Image Processing Using Hough Transforms, Proc. of the NASA/MPRIA Workshop: Pattern Recognition, Texas A&M University, College Station, Texas, February 3-4, 1983.

L. Davis, Fu-pei Hu, V. Hwang, and L. Kitchen, Image Matching Using Generalized Hough Transforms, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

D. Dow, Progress in the Scene-to-Map Registration Task, Proc. of the NASA/MPRIA Workshop: Pattern Recognition, Texas A&M University, College Station, Texas, February 3-4, 1983.

D. Dow, Progress in the Scene-to-Map Registration Investigation, Proc. of NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

L. F. Guseman, Jr., and L. Schumaker, Spline Classification Methods, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

D. Held, Reduction and Utilization of Speckle Noise in SAR Imagery, Proc. of the NASA/MPRIA Workshop: Pattern Recognition, Texas A&M University, College Station, Texas, February 3-4, 1983.

R. Heydorn, NASA Fundamental Research Program in MPRIA, Proc. of 15th Symposium on Interface for Computer Sciences and Statistics, March 1983.

R. Heydorn and R. Basu, Estimating Location Parameters in a Mixture Model, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

T. H. Joo and D. N. Held, SAR Speckle Noise Reduction Using Wiener Filter, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

Laveen N. Kanal, Subpixel Registration Accuracy and Modeling, Proc. of the NASA/MPRIA Workshop: Pattern Recognition, Texas A&M University, College Station, Texas, February 3-4, 1983.

H. Kostal, "Localized Shrinkage Factors and Minimax Results," in Proceedings of the NASA/MPRIA Workshop, L. Guseman, ed., Texas A&M University (1983) pp. 109-114.

H. Kostal, "Empirical Bayes Methods for Time and Spatial Series," contributed paper to the ASA Annual Meeting; Toronto, Canada, August 15-18, 1983.

David Lavine, Laveen Kanal, Carlos A. Berenstein, Analysis of Subpixel Registration Accuracy, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

E. M. Mikhail and Fidel D. Paderes, Jr., Aspects of Simulation for Rectification Studies, Proc. of the NASA/MPRIA Workshop: Pattern Recognition, Texas A&M University, College Station, Texas, February 3-4, 1983.

E. M. Mikhail and Fidel C. Paderes, Jr., "Simulation Aspects in the Study of Rectification of Satellite Scanner Data", Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

C. N. Morris, "A Minimax Approach to Spatial Estimation Using Affinity Matrices," in Proceedings of the NASA/MPRIA Workshop, L. Guseman, ed., Texas A&M University (1983) pp. 101-108.

C. N. Morris and H. Kostal, "An Empirical Bayes Approach to Spatial Analysis," in Proceedings of the NASA/MPRIA Symposium, L. Guseman, ed., Johnson Space Center, Houston, Texas (1983) pp. 143-165.

C. N. Morris, "Spatial Estimation from Remotely Sensed Data via Empirical Bayes Models," invited paper to be presented at the Computer Science and Statistics: 16th Symposium on the Interface, Image Processing Session; Atlanta, Georgia, March 14-16, 1984.

M. Naraghi, Autoregressive Models for Use in Scene Segmentation, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

M. Naraghi, Random Field Models for Use in Scene Segmentation, Proc. of the NASA/MPRIA Workshop: MATH/STAT, Texas A&M University, College Station, Texas, January 27-28, 1983

H. J. Newton, Confidence Bands for Autoregressive Spectra, Am. Stat. Assoc., National Meeting, Toronto, 1983.

F. C. Paderes, Jr., for E. M. Mikhail, "Registration/Rectification of Remotely Sensed Data," Program kickoff meeting for the Fundamental Research Program in Mathematical Pattern Recognition and Image Analysis, NASA Lunar Planetary Institute, Houston, Texas, August 10-11, 1982.

F. C. Paderes, Jr., and E. M. Mikhail, "Photogrammetric Aspects of Satellite Imageries," Proceedings of The American Society of Photogrammetry Fall convention, Salt Lake City, Utah, September 19-23, 1983.

F. C. Paderes, Jr., and E. M. Mikhail, "Rectification of Single and Overlapping Frames of Satellite Scanner Data," Paper to be presented at the XVth International Society for Photogrammetry and Remote Sensing Congress at Rio de Janeiro, Brazil, June 11-29, 1984.

E. Parzen, FUN.STAT and Statistical Image Representations, Proc. of the NASA/MPRIA Workshop: MATH/STAT, Texas A&M University, College Station, Texas, January 27-28, 1983.

E. Parzen, Statistical Image Representations: Non-Gaussian Classification, Proc. of the NASA/MPRIA Workshop: MATH/STAT, Texas A&M University, College Station, Texas, January 27-28, 1983.

E. Parzen, Quantile Data Analysis of Image Data, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

E. Parzen, Repeated Measures Analysis of Image Data, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

C. Peters, Consistency and Other Large Sample Properties of Maximum Likelihood Estimates of Mixture Parameters, Proc. of the NASA/MPRIA Workshop on Density Estimation, Texas A&M University, College Station, Texas, March 1982.

C. Peters and H. P. Decell, Jr., Co-variance Hypotheses of LANDSAT Data, Proc. of the NASA/MPRIA Workshop, MATH/STAT, Texas A&M University, College Station, Texas, January 1982.

C. Peters, Mixture Models for Dependent Observations, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

L. L. Schumaker, Second Edmonton Conference on Approximation Theory, On space of piecewise polynomials with boundary conditions, June 1982.

L. L. Schumaker, SIAM Stanford, Special session on surfaces, July 1982, Spaces of piecewise polynomials.

L. L. Schumaker, International Conference on Surface Fitting, Lake Garda, Italy, June 1983. Five one-hour lectures.

L. L. Schumaker, Conference on Numerical Analysis, University of Dundee, Scotland, June 1983. Computing the zeros of splines.

L. L. Schumaker, SIAM Computer Aided Design Conference, RFI, Troy, New York, June 1983, Surface fitting.

L. L. Schumaker, Fitting of Histograms, Numerical Analysis Conference, Texas Tech. University, Lubbock, Texas, September 1983.

L. L. Schumaker, NASA SRAEC Meeting, Colorado State University, Fort Collins, Colorado, January 9-11, 1984, Splines and applications.

D. W. Scott, Multivariable Density Estimation and Remote Sensing, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

D. W. Scott and J. R. Thompson, Probability Estimation in Higher Dimension, Proc. of 15th Symposium on Interface of Computer Science and Statistics, March 1983.

D. W. Scott, "Review of Some Results in Bivariate Density Estimation." Proc. of NASA Workshop on Density Estimation and Function Smoothing, pp. 165-194, March 1982.

D. W. Scott, "Optimal Meshes for Histograms Using Variable-Width Bins," poster session, ASA Annual Meeting, Cincinnati, Ohio, August 16-19, 1982.

D. W. Scott, Average Shifted Histograms: Effective Nonparametric Estimation, Rice Technical Report.

K. M. Shanmugan, Textural Edge Detection and Sensitivity Analysis, Proc. of the NASA/MPRIA Workshop: Pattern Recognition, Texas A&M University, College Station, Texas, February 3-4, 1983.

K. M. Shanmugan, "A Frequency Domain Model for Markov Texture Fields," presented at the 1983 Systems, Man and Cybernetics Conference, New Delhi, India, December 27, 1983 - January 7, 1984.

Grahame Smith, Approaches to Image Registration and Segmentation, Proc. of the NASA/MPRIA Workshop: Pattern Recognition, Texas A&M University, College Station, Texas, February 3-4, 1983.

Grahame Smith, Shape from Shading: An Assessment, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

Grahame Smith, The Relationship Between Image Irradiance and Surface Orientation, presented at the Computer Vision and Pattern Recognition Conference, Washington, D.C., June 19-23, 1983.

W. B. Smith, Discrimination Relative to Measures of Non-Normality, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

A. H. Strahler, W. R. Tobler, and C. E. Woodcock, Improving Spatial Modeling in Remote Sensing, Proc. of the NASA/MPRIA Workshop: Pattern Recognition, Texas A&M University, College Station, Texas, February 3-4, 1983.

A. H. Strahler and C. E. Woodcock, Relating Spatial Patterns in Image Data to Scene Characteristics, Proc. of the NASA/MPRIA Symposium, NASA/Johnson Space Center, Houston, Texas, June 1-3, 1983.

J. R. Thompson and D. Scott, Nonparametric Probability Density Estimation for Data Analysis in Several Dimensions, Proc. of 27th Conference on Design of Experiments in Army Research Development and Testing, June 1983.

Miscellaneous Reports

R. S. Chhikara, and A. G. Houston, Estimation with Classifier as Auxiliary Variable, NASA/Lockheed Technical Report (being revised for submission to technical journal).

L. S. Davis, and L. Kitchen, Image matching using generalized Hough transforms, F. Hu, V. Hwang, University of Maryland, Center for Automation Research, TR-27, October 1983.

D. Harwood, M. Subbarao, and L. S. Davis, Texture classification by local rank correlation, University of Maryland, Computer Science, TR-1314, August 1983.